OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4729

Mechanics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4729
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

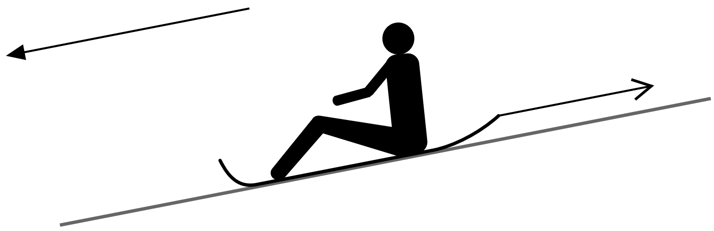
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1



A sledge with its load has mass 70 kg. It moves down a slope and the resistance to the motion of the sledge is 90 N. The speed of the sledge is controlled by the constant tension in a light rope, which is attached to the sledge and parallel to the slope (see diagram). While travelling 20 m down the slope, the speed of the sledge decreases from 2.1 m s^{-1} to 1.4 m s^{-1} and it descends a vertical distance of 3 m.

(i) Calculate the change in energy of the sledge and its load. **[4]**

(ii) Calculate the tension in the rope. **[3]**

2 A car of mass 1250 kg travels along a straight road inclined at 2° to the horizontal. The resistance to the motion of the car is kv N, where $v \text{ m s}^{-1}$ is the speed of the car and k is a constant. The car travels at a constant speed of 25 m s^{-1} up the slope and the engine of the car works at a constant rate of 21 kW.

(i) Calculate the value of k. **[4]**

(ii) Calculate the constant speed of the car on a horizontal road. **[3]**

3 A uniform lamina $ABCDE$ consists of a square $ACDE$ and an equilateral triangle ABC which are joined along their common edge AC to form a pentagon whose sides are each 8 cm in length.

(i) Calculate the distance of the centre of mass of the lamina from AC. **[5]**

(ii) The lamina is freely suspended from A and hangs in equilibrium. Calculate the angle that AC makes with the vertical. **[2]**

4 Two small spheres A and B are moving towards each other along a straight line on a smooth horizontal surface. A has speed 3 m s^{-1} and B has speed 1.5 m s^{-1} before they collide directly. The direction of motion of B is reversed in the collision. The speeds of A and B after the collision are 2 m s^{-1} and 2.9 m s^{-1} respectively.

(i) (a) Show that the direction of motion of A is unchanged by the collision. **[2]**

(b) Calculate the coefficient of restitution between A and B. **[2]**

The mass of B is 0.2 kg.

(ii) Find the mass of A. **[3]**

B continues to move at 2.9 m s^{-1} and strikes a vertical wall at right angles. The wall exerts an impulse of magnitude 0.68 N s on B.

(iii) Calculate the coefficient of restitution between B and the wall. **[4]**

5 A particle is projected with speed $7\,\mathrm{m\,s^{-1}}$ at an angle of elevation of $30°$ from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time t s are x m and y m respectively.

(i) Express x and y in terms of t and hence find the equation of the trajectory of the particle. **[4]**

(ii) Calculate the values of x when $y = 0.6$. **[4]**

(iii) Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. **[4]**

6

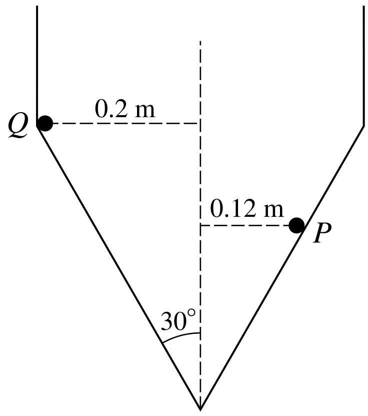


Fig. 1

A container is constructed from a hollow cylindrical shell and a hollow cone which are joined along their circumferences. The cylindrical shell has radius 0.2 m, and the cone has semi-vertical angle $30°$. Two identical small spheres P and Q move independently in horizontal circles on the smooth inner surface of the container (see Fig. 1). Each sphere has mass 0.3 kg.

(i) P moves in a circle of radius 0.12 m and is in contact with only the conical part of the container. Calculate the angular speed of P. **[5]**

(ii)

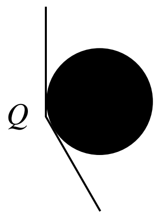


Fig. 2

Q moves with speed $2.1\,\mathrm{m\,s^{-1}}$ and is in contact with both the cylindrical and conical surfaces of the container (see Fig. 2). Calculate the magnitude of the force which the cylindrical shell exerts on the sphere. **[4]**

(iii) Calculate the difference between the mechanical energy of P and of Q. **[5]**

[Question 7 is printed overleaf.]

7

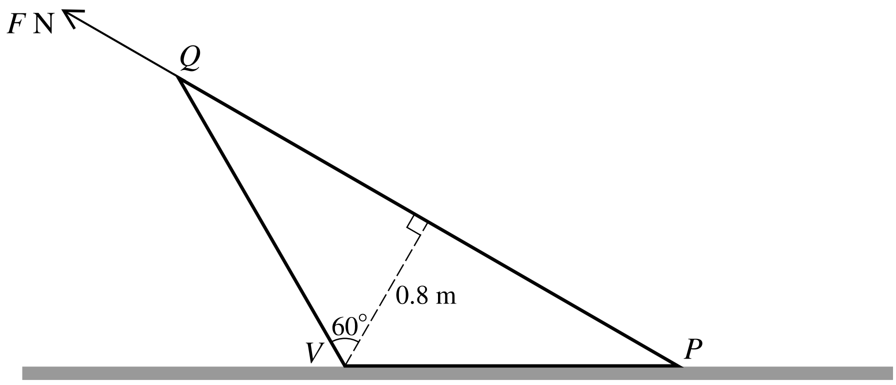


Fig. 1

A uniform solid cone of height 0.8 m and semi-vertical angle 60° lies with its curved surface on a horizontal plane. The point P on the circumference of the base is in contact with the plane. V is the vertex of the cone and PQ is a diameter of its base. The weight of the cone is 550 N. A force of magnitude F N and line of action PQ is applied to the base of the cone (see Fig. 1). The cone topples about V without sliding.

(i) Calculate the least possible value of F. **[4]**

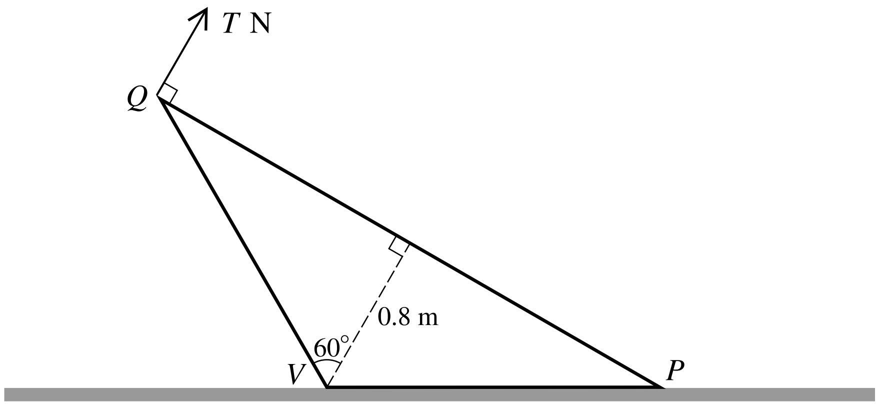


Fig. 2

The force of magnitude F N is removed and an increasing force of magnitude T N acting upwards in the vertical plane of symmetry of the cone and perpendicular to PQ is applied to the cone at Q (see Fig. 2). The coefficient of friction between the cone and the horizontal plane is μ.

(ii) Given that the cone slides before it topples about P, calculate the greatest possible value for μ. **[10]**

1 i	PE = 70x3g KE change =70x($2.1^2 - 1.4^2$)/2 PE change + KE change 2143.75 J	B1 B1 M1 A1 [4]	2058 85.75 Must include evaluation Accept 2140. Allow all values to be negative.
ii	 20(90 + T) = 2143.75 T = 17.1875 N	M1 A1ft A1 [3]	Work done = Energy change used ft(cv(2143.75)) accept 17.2
OR	 70g.0.15 – 90 – T = 70.(-0.06125) T = 17.1875 N	M1 A1 A1 [3]	Use of $v^2=u^2 + 2as$ to find a AND use of N2 law(4 terms) accept 17.2

2 i	21000/25 0 = 21000/25 – 25k – 1250gsin2 k = 16.5	B1 M1 A1 A1 [4]	Use of force = power/speed 3 terms cv(21000/25)
ii	 21000/v= 16.5v v = 35.7 ms^{-1}	M1 A1ft A1 [3]	 ft on cv(k)

3 i	 - (8cos30/3)(8^2sin60/2) + (4)(8^2) = (8^2 +8^2sin60/2)(x_G) x_G = 2.09 cm	M1 A1 A1 A1 A1 [5]	Table of moments idea, may include g and/or density. -2.309 x 27.7
ii	tanθ = (2.09/4) θ = 27.6°	M1 A1ft [2]	 ft cv(x_G)

4 ia b	If reversed 2.9 + 2 = e(3 + 1.5) e > 1 impossible 2.9 - 2 = e(3 + 1.5) e = 0.2	M1 A1 [2] M1 A1 [2]	 Award B1 if no explicit numerical justification May be seen in ia
ii	 3m – 0.2x1.5 = 2m + 0.2x2.9 m = 0.88	M1 A1 A1 [3]	Conservation of momentum Accept with g included consistently Do not award if g used
iii	0.68 = 0.2v + 0.2x2.9 v = 0.5 e = 0.5/2.9 e = 0.172	M1 A1 M1 A1 [4]	Impulse = change in momentum Separation speed not 2.9 Allow 5/29

5 i	x = (7cos30)t y = (7sin30)t – $gt^2/2$ y = xtan30 - $gx^2/(2x7^2\cos^2 30)$	B1 B1 M1 A1 [4]	 Attempt to eliminate t y = x/√3 – $2x^2/15$ or y = 0.577x – $0.133x^2$ aef
ii	 $2x^2/15$ – x/√3 + 0.6 = 0 or $9.8t^2$ – 7t + 1.2 = 0 x = 1.73 m or √3 m 2.6(0) m or 3√3/2 m	M1 M1 A1 A1 [4]	Create a 3 term Q.E. in x or t with y = 0.6 Solve 3 term Q.E. for x or t
iii **OR**	$v^2 = (7\sin 30)^2 – 2x9.8x0.6$ v = 0.7 ms^{-1} tanθ = 0.7/(7cos30) θ = 6.59° to horizontal or 83.4° to vertical Attempt to differentiate equation of trajectory tan30 - $gx/(7^2\cos^2 30)$ Substitute x = √3 and equate to tanθ θ = 6.59° to horizontal or 83.4° to vertical	M1 A1 M1 A1 [4] M1 A1 M1 A1 [4]	Using $v^2 = u^2$ -2gs with u a component of 7; can find t first from their x in (i), and then use v = u + at. Use component of 7 Allow 1/√3 – 4x/15 or y' = 0.577 – 0.267x

6		M1	
i	Rsin30 = 0.3g	A1	R = 5.88 or 0.6g
		M1	
	Rcos30 = $0.3\omega^2$ x 0.12	A1	accept $v^2/0.12$ for acceleration
	ω = 11.9 $rads^{-1}$	A1	cao
		[5]	
ii		M1	Resolve and use N2L on sphere Q, 3 terms needed
	S + Rcos30 = $0.3x2.1^2/0.2$	A1	
	R = 5.88	B1ft	ft cv(R) from (i)
	S = 1.52 N	A1	
		[4]	
iii	v_P = 11.9x0.12, or h = 0.2/tan30 or 0.12/tan30 or 0.08/tan30	B1	cv(ω) from (i)
	+/-(Q – P) =	M1	Attempt to calculate KE or PE for both particles
	$0.5x0.3(2.1^2 – (11.9x0.12)^2)$	A2ft	KE difference (ft on cv(ω)) or PE difference
	+ (0.2/tan30 – 0.12/tan30) x 0.3g		
	Q-P = +/- 0.763 J	A1	Q – P = +/-(0.3556 + 0.4074)
		[5]	

7		M1	Attempt at moments
i	F x 0.8 =	A1	
	0.6cos60 x 550	A1	
	F = 206.25	A1	Accept 206, cao
		[4]	
ii		M1*	Moment of T about P
	T x 2 x 0.8/tan30	A1	T x 2.77
	=	M1*	Moment of weight about P
	550 x (0.8/sin30 – 0.6cos60)	A1	550 x (1.6 – 0.3)
	T = 258	A1	Accept to 2sf
		M1*	Resolving vertically, 3 terms needed
	R = 550 – Tcos30	A1	Value for T not required
	Fr = Tsin30	B1*	Value for T not required; accept < or ≤
	μ = 129/326.6	M1dep*	For correct use of F = μR, R ≠ 550
	μ = 0.395	A1	
		[10]	

OR	$T \times 0.8/\tan30 + 550 \times 0.6\cos60 = R \times 0.8/\cos60$	M1* A2	Moments about V, 3 terms needed A1 for two terms correct
		M1*	Resolving vertically, 3 terms needed
	$R = 550 – T\cos30$	A1	
	Solve for T or R	M1	
	$T = 258$ or $R=326.5625$	A1	Accept to 2sf
	$Fr = T\sin30$	B1*	Value for T not required; accept < or ≤
	$\mu = 129/326.6$	M1dep*	For correct use of $F = \mu R$, $R \neq 550$
	$\mu = 0.395$	A1 [10]	
OR	$Fr \times 1.6\cos30 + 550 \times (1.6\sin30 +0.6\sin30) = R \times (1.6 + 1.6\sin30)$	M1* A2	Moments about Q, 3 terms needed A1 for two terms correct
		M1*	Resolving vertically, 3 terms needed
	$R = 550 – T\cos30$	A1	
	$Fr = T\sin30$	B1*	accept < or ≤
	Solving for at least one of R, Fr, or T	M1	
	Either $R = 326.5625$, or $Fr = 129(.0017008)$, or $T=258$	A1	Only one needed. Accept to 2sf.
	$\mu = 129/326.6$	M1dep*	For correct use of $F = \mu R$, $R \neq 550$
	$\mu = 0.395$	A1 [10]	